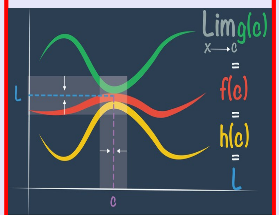
Calculus I

Lecture 13



Feb 19-8:47 AM

Class Quiz 13

Box Your Final Answers.

Given $f(x) = \boxed{x \sin x} + \frac{\pi}{2}$

1) Find $f(\frac{\pi}{2}) = \frac{\pi}{2} \cancel{\sin\frac{\pi}{2}}^{1} + \frac{\pi}{2}$

$= \frac{\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{2} = \boxed{\pi}$

2) Find $f'(x) = 1 \cdot \sin x + x \cdot \cos x + 0$

$= \boxed{\sin x + x\cos x}$

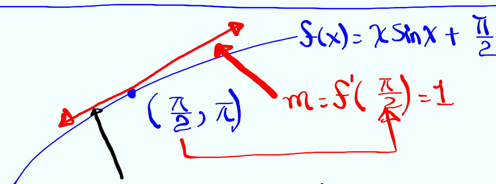
3) Find $f'(\frac{\pi}{2}) = \cancel{\sin\frac{\pi}{2}}^{1} + \frac{\pi}{2}\cancel{\cos\frac{\pi}{2}}^{0}$

$= 1 + \frac{\pi}{2} \cdot 0 = \boxed{1}$

Hint:

$\frac{d}{dx}[f \cdot g] = f'g + fg'$

$\frac{d}{dx}[\sin x] = \cos x$



$y - y_1 = m(x - x_1)$

$y - \pi = 1(x - \frac{\pi}{2})$

$y = x - \frac{\pi}{2} + \pi \longrightarrow \boxed{y = x + \frac{\pi}{2}}$

Mar 26-8:01 AM

Find $f'(x)$

1) $f(x) = \sqrt{x}\,(x-1)$

$$f'(x) = \frac{d}{dx}[\sqrt{x}]\,(x-1) + \sqrt{x}\,\frac{d}{dx}[x-1]$$

$$= \frac{1}{2\sqrt{x}}(x-1) + \sqrt{x}\cdot 1$$

$$= \frac{1}{2\sqrt{x}}\cdot x - \frac{1}{2\sqrt{x}} + \sqrt{x}$$

$$= \frac{1}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} + \sqrt{x}$$

$$= \boxed{\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}}$$

2) $f(x) = \dfrac{x^3}{x^2-1}$

$$f'(x) = \frac{\frac{d}{dx}[x^3](x^2-1) - x^3\cdot\frac{d}{dx}[x^2-1]}{(x^2-1)^2}$$

$$= \frac{3x^2(x^2-1) - x^3\cdot 2x}{(x^2-1)^2} = \frac{3x^4-3x^2-2x^4}{(x^2-1)^2} = \boxed{\frac{x^4-3x^2}{(x^2-1)^2}}$$

Mar 26-9:06 AM

Find $f'(x)$, then Solve $f'(x)=0$

1) $f(x) = (x^2+3x-1)(x^2-5x)$

$$f'(x) = (2x+3)(x^2-5x) + (x^2+3x-1)\cdot(2x-5)$$

$$= 2x^3 - 10x^2 + 3x^2 - 15x + 2x^3 - 5x^2 + 6x^2 - 15x - 2x + 5$$

$$\boxed{f'(x) = 4x^3 - 6x^2 - 32x + 5}$$

Polynomial

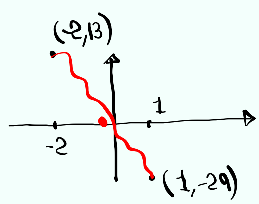
Cont. $(-\infty, \infty)$

To Solve $f'(x)=0$ we may have to use technology or Synthetic Div.

$$\begin{array}{r|rrrr} 1 & 4 & -6 & -32 & 5 \\ & & 4 & -2 & -34 \\ \hline & 4 & -2 & -34 & -29 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 4 & -6 & -32 & 5 \\ & & 12 & 18 & -42 \\ \hline & 4 & 6 & -14 & -37 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & -6 & -32 & 5 \\ & & -8 & 28 & 8 \\ \hline & 4 & -14 & -4 & 13 \end{array}$$



By I.V.T. $f(x)=0$ for some number c in $(-2,1)$

Mar 26-9:19 AM

2) $f(x) = \dfrac{x-3}{x+3}$

$$f'(x) = \frac{1(x+3)-(x-3)\cdot 1}{(x+3)^2} \qquad f'(x) = \frac{6}{(x+3)^2}$$

$f'(x) = 0 \qquad \dfrac{6}{(x+3)^2} = 0 \rightarrow 6 = 0$

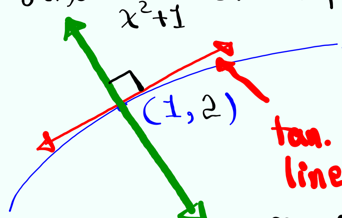
False

$f'(x) \neq 0$

$f'(x) > 0 \qquad \dfrac{+}{+} > 0$

Mar 26-9:34 AM

Find equation of the $\boxed{\text{normal line}}$ to the graph of $f(x) = \dfrac{3x+1}{x^2+1}$ at the point with $x = 1$.



$f(x) = \dfrac{3x+1}{x^2+1}$

$f(1) = \dfrac{3+1}{1+1} = 2$

$m_{\text{tan. line}} = f'(1)$

$m_1 \cdot m_2 = -1$

$m_{\text{normal line}} = \dfrac{-1}{m_{\text{tan. line}}}$

$f(x) = \dfrac{3x+1}{x^2+1}$

$f'(x) = \dfrac{3(x^2+1)-(3x+1)\cdot 2x}{(x^2+1)^2}$

$f'(1) = \dfrac{3(1+1)-(3+1)\cdot 2}{(1+1)^2}$

$= \dfrac{6-8}{4} = \dfrac{-2}{4} = \dfrac{-1}{2}$

$m_{\text{Normal Line}} = \dfrac{-1}{-\frac{1}{2}} = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x-1)$

$y = 2x - 2 + 2$

$\boxed{y = 2x}$

Mar 26-9:38 AM

Find $f'(x)$ & $f''(x)$ for $f(x) = \frac{1}{x-2}$.

$$f'(x) = \frac{0\cdot(x-2) - 1\cdot 1}{(x-2)^2} = \frac{-1}{(x-2)^2} = \frac{-1}{x^2-4x+4}$$

$$f''(x) = \frac{0\cdot(x^2-4x+4) - (-1)\cdot(2x-4)}{(x^2-4x+4)^2}$$

$$f''(x) = \frac{2x-4}{[(x-2)^2]^2} = \frac{2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}}$$

$$= \boxed{\frac{2}{(x-2)^3}}$$

Mar 26-9:49 AM

Chain Rule

If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

ex: $y = \sin x^2$

$u = x^2 \longrightarrow \frac{du}{dx} = 2x$

$y = \sin u$

$y' = \cos u \cdot 2x$

$y' = \cos x^2 \cdot 2x$

$\boxed{y' = 2x \cos x^2}$

Mar 26-10:24 AM

$y = \sqrt{\cos x}$

$u = \cos x$

$y = \sqrt{u}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{1}{2\sqrt{u}} \cdot (-\sin x)$

$\boxed{y' = \frac{-\sin x}{2\sqrt{\cos x}}}$

$y = (x^3 + \tan x)^2$

$u = x^3 + \tan x$

$y = u^2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 2u \cdot (3x^2 + \sec^2 x)$

$y' = 2(x^3 + \tan x)(3x^2 + \sec^2 x)$

Mar 26-10:28 AM

Power Rule & Chain Rule

$$\frac{d}{dx}[u^n] = n\,u^{n-1}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[(f(x))^n\right] = n\,[f(x)]^{n-1}\,\boxed{\frac{d}{dx}[f(x)]}$$

$f(x) = (x^3 + 3x^2 - 8x)^{10}$

$f'(x) = 10(x^3 + 3x^2 - 8x)^9 \cdot (3x^2 + 6x - 8)$

$f(x) = \sin^3 x^5$ $\quad$ $f(x) = [\sin x^5]^3$

$f'(x) = 3[\sin x^5]^2 \cdot \cos x^5 \cdot 5x^4$

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$f(x) = \cos(\tan\sqrt{x})$

$f'(x) = -\sin(\tan\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$f(x) = \left(\frac{x+4}{x-6}\right)^5$

$f'(x) = 5\left(\frac{x+4}{x-6}\right)^4 \cdot \frac{\overbrace{1(x-6)-(x+4)\cdot 1}^{-10}}{(x-6)^2}$

$= 5\left(\frac{x+4}{x-6}\right)^4 \cdot \frac{-10}{(x-6)^2}$

$= \boxed{\frac{-50(x+4)^4}{(x-6)^6}}$

Mar 26-10:40 AM

find $f'(x)$

1) $f(x) = (x^2-4x)^{100}$

$f'(x) = 100(x^2-4x)^{99} \cdot (2x-4)$

$= 200(x^2-4x)^{99}(x-2)$

2) $f(x) = \frac{1}{(1+\sec x)^8}$ $\quad f(x) = (1+\sec x)^{-8}$

$f'(x) = -8(1+\sec x)^{-9} \cdot (\sec x \tan x)$

$\boxed{f'(x) = \frac{-8\sec x \tan x}{(1+\sec x)^9}}$

$f'(x) = \frac{0(\cancel{1+\sec x})^8 - 1 \cdot 8(1+\sec x)^7 \cdot \sec x \tan x}{[(1+\sec x)^8]^2} = \frac{-8(\cancel{1+\sec x})^7 \cdot \sec x \tan x}{(\cancel{1+\sec x})^{16}}$ $\quad 0$ $\quad (1+\sec x)^9$

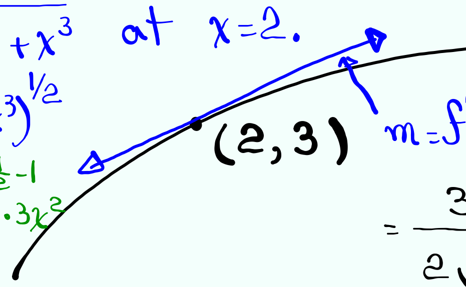
Mar 26-10:47 AM

Find eqn of tan. line to the graph of

$f(x) = \sqrt{1+x^3}$ at $x = 2$.

$f(x) = (1+x^3)^{1/2}$

$f'(x) = \frac{1}{2}(1+x^3)^{\frac{1}{2}-1} \cdot 3x^2$



$m = f'(2)$

$= \frac{3 \cdot 2^2}{2\sqrt{1+2^3}}$

$f'(x) = \frac{3x^2}{2(1+x^3)^{1/2}}$ $\quad f'(x) = \frac{3x^2}{2\sqrt{1+x^3}}$ $\quad = \frac{12}{2 \cdot 3} = \boxed{2}$

$y - 3 = 2(x-2)$

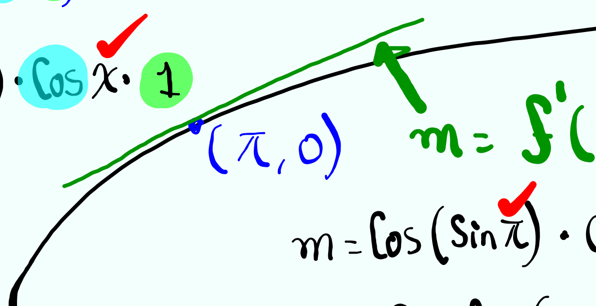
$\longrightarrow \boxed{y = 2x - 1}$

Mar 26-10:58 AM

Find eqn of tan. line to the graph of

$f(x) = \sin(\sin x)$ at $x = \pi$.

$f'(x) = \cos(\sin x) \cdot \cos x \cdot 1$



$m = f'(\pi)$

$m = \cos(\sin \pi) \cdot \cos \pi$

$= \cos 0 \cdot \cos \pi$

$= 1 \cdot (-1) = -1$

$y - 0 = -1(x - \pi)$

$\boxed{y = -x + \pi}$

Mar 26-11:05 AM

$\boxed{\text{Find}}$ all x-coordinates of $\boxed{\text{all points}}$ where $f(x) = \sin 2x - 2\sin x$ have horizontal tan. line

$m = 0$

Solve $f'(x) = 0$

$f'(x) = \cos 2x \cdot 2 - 2 \cdot \cos x$

Solve

$$2\cos 2x - 2\cos x = 0$$

$$\cos 2x - \cos x = 0$$

$$2\cos^2 x - 1 - \cos x = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$2\cos x + 1 = 0$ | $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ | $\cos x = 1$

Ref. Angle $\frac{\pi}{3}$ | 0

QII QIII

$\pi - \frac{\pi}{3}$ $\quad \pi + \frac{\pi}{3}$

$\boxed{\frac{2\pi}{3} + 2n\pi}$ $\boxed{\frac{4\pi}{3} + 2n\pi}$ $\quad 0 + \boxed{2n\pi}$

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